Cambridge International AS & A Level

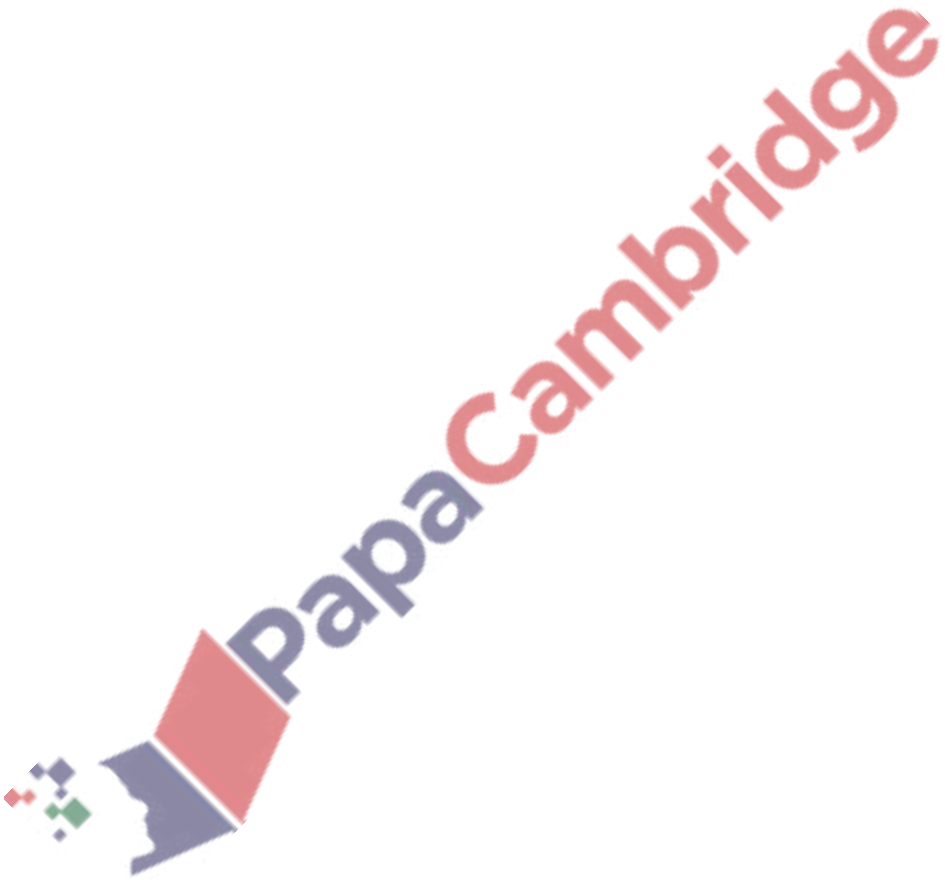
MATHEMATICS (9709) P2

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS

PapaCambridge

Chapter 1

Algebra



1. 9709_s20_qp_21 Q: 2

The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = 6x^3 + ax^2 + 9x + b,$$

where a and b are constants. It is given that $(x - 2)$ and $(2x + 1)$ are factors of $\mathrm{p}(x)$.

Find the values of a and b. [5]

2. 9709_s20_qp_21 Q: 4

(a) Sketch, on the same diagram, the graphs of $y = |3x + 2a|$ and $y = |3x - 4a|$, where a is a positive constant.

Give the coordinates of the points where each graph meets the axes. [3]

(b) Find the coordinates of the point of intersection of the two graphs. [3]

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(c) Deduce the solution of the inequality $|3x + 2a| < |3x - 4a|$. [1]

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3. 9709_s20_qp_22 Q: 5

(a) Sketch, on the same diagram, the graphs of $y = |2x - 3|$ and $y = 3x + 5$. [2]

(b) Solve the inequality $3x + 5 < |2x - 3|$. [3]

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4. 9709_w20_qp_21 Q: 1

Given that

$$\ln(2x + 1) - \ln(x - 3) = 2,$$

find x in terms of e. [4]

5. 9709_w20_qp_21 Q: 2

The polynomial p(x) is defined by

$$\mathrm{p}(x) = x^3 + ax^2 + bx + 16,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of p(x) and that the remainder is 72 when p(x) is divided by $(x - 2)$.

Find the values of a and b. [5]

6. 9709_w20_qp_21 Q: 4

(a) Solve the equation $|2x - 5| = |x + 6|$. [3]

(b) Hence find the value of y such that $\left|2^{1-y} - 5\right| = \left|2^{-y} + 6\right|$. Give your answer correct to 3 significant figures. [2]

7. 9709_w20_qp_22 Q: 3

(a) Sketch, on a single diagram, the graphs of $y = \left|\frac{1}{2}x - a\right|$ and $y = \frac{3}{2}x - \frac{1}{2}a$, where a is a positive constant. [2]

(b) Find the coordinates of the point of intersection of the two graphs. [3]

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(c) Deduce the solution of the inequality $\left|\frac{1}{2}x - a\right| > \frac{3}{2}x - \frac{1}{2}a$. [1]

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8. 9709_m19_qp_22 Q: 2

Given that x satisfies the equation $|2x + 3| = |2x - 1|$, find the value of

$$|4x - 3| - |6x|.$$ [4]

9. 9709_m19_qp_22 Q: 4

(i) Find the quotient when $4x^3 + 8x^2 + 11x + 9$ is divided by $(2x + 1)$, and show that the remainder is 5. [3]

ii) Show that the equation $4x^3 + 8x^2 + 11x + 4 = 0$ has exactly one real root. [3]

10. 9709_s19_qp_21 Q: 2

(i) Solve the inequality $|3x - 5| < |x + 3|$. [4]

(ii) Hence find the greatest integer n satisfying the inequality $|3^{0.1n+1} - 5| < |3^{0.1n} + 3|$. [2]

11. 9709_s19_qp_21 Q: 5

The polynomial p(x) is defined by

$$\mathrm{p}(x) = 5x^3 + ax^2 + bx - 16,$$

where a and b are constants. It is given that $(x - 2)$ is a factor of p(x) and that the remainder is 27 when p(x) is divided by $(x + 1)$.

(i) Find the values of a and b. [5]

(ii) Hence factorise p(x) completely. [3]

12. 9709_s19_qp_22 Q: 1

The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = 4x^3 + (k+1)x^2 - mx + 3k,$$

where k and m are constants. Given that $(x+1)$ is a factor of $\mathrm{p}(x)$, express m in terms of k. [3]

13. 9709_s19_qp_22 Q: 2

(i) Solve the equation $|4 + 2x| = |3 - 5x|$. [3]

(ii) Hence solve the equation $|4 + 2e^{3y}| = |3 - 5e^{3y}|$, giving the answer correct to 3 significant figures. [2]

14. 9709_w19_qp_21 Q: 1

(i) Solve the inequality $|2x - 7| < |2x - 9|$. [3]

(ii) Hence find the largest integer n satisfying the inequality $|2\ln n - 7| < |2\ln n - 9|$. [2]

15. 9709_w19_qp_21 Q: 4

The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = ax^3 + ax^2 - 15x - 18,$$

where a is a constant. It is given that $(x - 2)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a. [2]

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(ii) Using this value of a, factorise $\mathrm{p}(x)$ completely. [3]

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(iii) Hence solve the equation $\mathrm{p}(e^{\sqrt{y}}) = 0$, giving the answer correct to 2 significant figures. [2]

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16. 9709_w19_qp_22 Q: 1

The polynomial $f(x)$ is defined by

$$f(x) = x^4 - 3x^3 + 5x^2 - 6x + 11.$$

Find the quotient and remainder when $f(x)$ is divided by $(x^2 + 2)$. [3]

17. 9709_m18_qp_22 Q: 1

Solve the inequality $|5x + 2| > |4x + 3|$. [4]

18. 9709_m18_qp_22 Q: 4

The polynomial p(x) is defined by

$$p(x) = 4x^3 + 4x^2 - 29x - 15.$$

(i) Use the factor theorem to show that $(x + 3)$ is a factor of p(x). [2]

(ii) Factorise p(x) completely. [3]

(iii) Hence, given that

$$2^{3u+2} + 4^{u+1} = 29 \times 2^u + 15,$$

find the value of 2^u and, using logarithms, find the value of u correct to 3 significant figures. [3]

19. 9709_s18_qp_21 Q: 6

The cubic polynomial $f(x)$ is defined by

$$f(x) = x^3 + ax^2 + 14x + a + 1,$$

where a is a constant. It is given that $(x + 2)$ is a factor of $f(x)$.

(i) Use the factor theorem to find the value of a and hence factorise $f(x)$ completely. [5]

(ii) Hence, without using a calculator, solve the equation $f(2x) = 3f(x)$. [4]

20. 9709_s18_qp_22 Q: 1

Solve the inequality $|3x - 2| < |x + 5|$. [4]

21. 9709_s18_qp_22 Q: 3

(i) Find the quotient when

$$x^4 - 2x^3 + 8x^2 - 12x + 13$$

is divided by $x^2 + 6$ and show that the remainder is 1. [3]

(ii) Show that the equation

$$x^4 - 2x^3 + 8x^2 - 12x + 12 = 0$$

has no real roots. [3]

22. 9709_w18_qp_22 Q: 1

Solve the inequality $|3x - 5| < 2|x|$. [4]

23. 9709_m17_qp_22 Q: 6

The polynomial p(x) is defined by

$$\mathrm{p}(x) = ax^3 + bx^2 - 17x - a,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of p(x) and that the remainder is 28 when p(x) is divided by $(x - 2)$.

(i) Find the values of a and b. [5]

(ii) Hence factorise $p(x)$ completely. [3]

(iii) State the number of roots of the equation $p(2^y) = 0$, justifying your answer. [2]

24. 9709_s17_qp_21 Q: 2

Solve the inequality $|4 - x| \leqslant |3 - 2x|$. [4]

25. 9709_s17_qp_22 Q: 1

Solve the equation $|x + a| = |2x - 5a|$, giving x in terms of the positive constant a. [3]

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26. 9709_s17_qp_22 Q: 6

(i) Use the factor theorem to show that $(x + 2)$ is a factor of the expression

$$6x^3 + 13x^2 - 33x - 70$$

and hence factorise the expression completely. [5]

(ii) Deduce the roots of the equation

$$6 + 13y - 33y^2 - 70y^3 = 0.$$ [2]

27. 9709_w17_qp_21 Q: 5

The polynomial p(x) is defined by

$$\mathrm{p}(x) = ax^3 + bx^2 + 37x + 10$$

where a and b are constants. It is given that $(x + 2)$ is a factor of p(x). It is also given that the remainder is 40 when p(x) is divided by $(2x - 1)$.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

28. 9709_w17_qp_22 Q: 2

It is given that x satisfies the equation $|x+1|=4$. Find the possible values of

$$|x+4|-|x-4|. \qquad [4]$$

29. 9709_w17_qp_22 Q: 4

The polynomials $p(x)$ and $q(x)$ are defined by

$$p(x) = x^3 + x^2 + ax - 15 \qquad \text{and} \qquad q(x) = 2x^3 + x^2 + bx + 21,$$

where a and b are constants. It is given that $(x + 3)$ is a factor of $p(x)$ and also of $q(x)$.

(i) Find the values of a and b. [3]

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(ii) Show that the equation $q(x) - p(x) = 0$ has only one real root. [4]

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30. 9709_m16_qp_22 Q: 1

Find the quotient and the remainder when $2x^3 + 3x^2 + 10$ is divided by $(x + 2)$. [3]

31. 9709_m16_qp_22 Q: 2

Solve the inequality $|x - 5| < |2x + 3|$. [4]

32. 9709_s16_qp_21 Q: 4

The polynomial $p(x)$ is defined by

$$p(x) = 8x^3 + 30x^2 + 13x - 25.$$

(i) Find the quotient when $p(x)$ is divided by $(x + 2)$, and show that the remainder is 5. [3]

(ii) Hence factorise $p(x) - 5$ completely. [3]

(iii) Write down the roots of the equation $p(|x|) - 5 = 0$. [1]

33. 9709_s16_qp_22 Q: 2

(i) Find the quotient and remainder when $2x^3 - 7x^2 - 9x + 3$ is divided by $x^2 - 2x + 5$. [3]

(ii) Hence find the values of the constants p and q such that $x^2 - 2x + 5$ is a factor of $2x^3 - 7x^2 + px + q$. [2]

34. 9709_s16_qp_22 Q: 3

(i) Solve the equation $|3u + 1| = |2u - 5|$. [3]

(ii) Hence solve the equation $|3\cot x + 1| = |2\cot x - 5|$ for $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [2]

35. 9709_w16_qp_22 Q: 1

Solve the equation $|0.4x - 0.8| = 2$. [3]

36. 9709_w16_qp_22 Q: 4

The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = 4x^3 + ax^2 + ax + 4,$$

where a is a constant.

(i) Use the factor theorem to show that $(x + 1)$ is a factor of $\mathrm{p}(x)$ for all values of a. [2]

(ii) Given that the remainder is -42 when $\mathrm{p}(x)$ is divided by $(x - 2)$, find the value of a. [2]

(iii) When a has the value found in part **(ii)**, factorise $\mathrm{p}(x^2)$ completely. [4]

37. 9709_w16_qp_23 Q: 4

The polynomial $p(x)$ is defined by

$$p(x) = ax^3 + 3x^2 + 4ax - 5,$$

where a is a constant. It is given that $(2x - 1)$ is a factor of $p(x)$.

(i) Use the factor theorem to find the value of a. [2]

(ii) Factorise $p(x)$ and hence show that the equation $p(x) = 0$ has only one real root. [4]

(iii) Use logarithms to solve the equation $p(6^y) = 0$ correct to 3 significant figures. [2]

38. 9709_s15_qp_21 Q: 4

The polynomials $f(x)$ and $g(x)$ are defined by

$$f(x) = x^3 + ax^2 + b \quad \text{and} \quad g(x) = x^3 + bx^2 - a,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $f(x)$. It is also given that, when $g(x)$ is divided by $(x + 1)$, the remainder is -18.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the greatest possible value of $g(x) - f(x)$ as x varies. [2]

39. 9709_s15_qp_22 Q: 2

(i) Given that $(x+2)$ is a factor of

$$4x^3 + ax^2 - (a+1)x - 18,$$

find the value of the constant a. [3]

(ii) When a has this value, factorise $4x^3 + ax^2 - (a+1)x - 18$ completely. [3]

40. 9709_w15_qp_21 Q: 6

(i) Find the quotient and remainder when

$$x^4 + x^3 + 3x^2 + 12x + 6$$

is divided by $(x^2 - x + 4)$. [4]

(ii) It is given that, when

$$x^4 + x^3 + 3x^2 + px + q$$

is divided by $(x^2 - x + 4)$, the remainder is zero. Find the values of the constants p and q. [2]

(iii) When p and q have these values, show that there is exactly one real value of x satisfying the equation

$$x^4 + x^3 + 3x^2 + px + q = 0$$

and state what that value is. [3]

41. 9709_w15_qp_23 Q: 4

(i) Find the quotient when $3x^3 + 5x^2 - 2x - 1$ is divided by $(x - 2)$, and show that the remainder is 39. [4]

(ii) Hence show that the equation $3x^3 + 5x^2 - 2x - 40 = 0$ has exactly one real root. [3]